Time-periodic Heating of a Rotating Horizontal Fluid Layer in a Vertical Magnetic Field

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Thermal instability in a horizontal layer of an electrically conducting fluid heated from below has been investigated under the effects of uniform rotation about a vertical axis and an applied uniform vertical magnetic field. The temperature field between the walls of the fluid layer consists of two parts; a steady part and a time-dependent part, which varies periodically. The effect of modulation of the walls temperature on the onset of convection has been studied using Floquets theory. Stabilizing and destabilizing effects on the onset of convective instability have been found. Some comparisons have been made. – 2000 Mathematics Subject Classification: 76E06, 76R10.

Key words: Thermal Instability; Modulation; Rayleigh Number; Rotation; Magnetic Field.

1. Introduction

In many astrophysical and geophysical contexts one encounters situations in which rotation and a magnetic field are simultaneously present. For example, convection in the Earth's metallic core and stellar interiors often occurs in the presence of strong rotational and magnetic constraints. Therefore in this paper we seek to understand how the effects of rotation and a uniform vertical magnetic field influence the thermal convection in a horizontal layer of an electrically conducting fluid that is heated from below and cooled from above.

Although the field is very interesting, only limited literature is available on rotating magnetoconvection (RMC). The first theoretical investigations on RMC in a horizontal fluid layer were carried out by Chandrasekhar [1, 2]. Later, Nakagawa [3, 4] in his experimental studies determined the variation of the critical Rayleigh number *Rc* in liquid mercury as a function of the rotation rate and magnetic field strength, and confirmed the theoretical prediction of Chandrasekhar [5] that, at large values of the Taylor number T and the Chandrasekhar number Q, the critical Rayleigh number *Rc* has a local minimum in the regime where $Q \sim T^{1/2}$. In theoretical studies of RMC, Eltayeb [6, 7] found that, for large T and Q, the magnetic scaling law $Rc \sim$ Q holds when $Q \gg T^{1/2}$, while the rotational scaling law $Rc \sim T^{3/2}$ holds when $T^{1/2} \gg Q^{3/2}$. In the range where the Lorentz and Coriolis forces are comparable, $Q < T^{1/2} < Q^{3/2}$, the critical Rayleigh number is reduced and varies as $Rc \sim T^{1/2} \sim T/Q$ [7]. Gupta et al. [8] investigated the problem under the joint influence of a rotation and magnetic field especially with a view to drive the bounds for the complex growth rate for an arbitrary oscillatory perturbation which may be neutral or unstable. Cattaneo and Hughes [9] studied the thermal instability in a layer of an uniform magnetic field. Applying rotation they discuss various modes of convective instability.

Dhiman Joginder [10] studied the thermal instability in a rotatory hydromagnetic fluid layer and found that the Benard convection can be suppressed by simultaneously applying a vertical uniform rotation and uniform vertical magnetic field, in a certain parametric range. Aurnou and Olson [11] found experimentally that for rotating magnetoconvection, convective heat transfer is inhibited by rotation. They found that, at low rotation rate, the critical Rayleigh number increases linearly with the magnetic field.

Many researchers have under different conditions investigated the thermal instability in a horizontal fluid layer with temperature modulation, but in the absence of a magnetic field and rotation. Some of them are Gershuni and Zhukhovitskii [12], Venezian [13], Rosenblat and Tanaka [14], Finucane and Kelly [15], Roppo et al. [16], Bhatia and Bhadauria [17], and Bhadauria and Bhatia [18].

The objective of the present paper is to find the critical conditions under which convection occurs in the presence of an uniform vertical rotation and an applied uniform vertical magnetic field. The effect of modulation of the walls' temperature on the thermal instability has been investigated. The walls temperature has been modulated using a sinusoidal profile. The results have been obtained for the following three cases: (a) when the plate temperatures are modulated in phase, (b) when the modulation is out of phase, and (c) when only the temperature of the lower plate is modulated, while that of the upper plate is held constant. The results have been compared with other theoretical results.

2. Formulation

Consider a fluid layer of a viscous, incompressible, magnetic fluid, confined between two parallel horizontal walls, a distance d apart. The walls are infinitely extended and rigid. The configuration is shown in Figure 1. The governing equations in the Boussinesq approximation are

$$\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\nabla)\boldsymbol{V} - \frac{\mu}{4\pi\rho_0}\boldsymbol{H}\nabla\boldsymbol{H} = -\frac{1}{\rho_0}\nabla p + \left(1+\frac{\delta\rho}{\rho_0}\right)\boldsymbol{X} - 2\boldsymbol{\Omega}\times\boldsymbol{V} + \nu\nabla^2\boldsymbol{V}, \quad (2.1)$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{V}\nabla)T = \kappa\nabla^2 T, \quad (2.2)$$

$$\frac{\partial \boldsymbol{H}}{\partial t} + (\boldsymbol{V}\nabla)\boldsymbol{H} - (\boldsymbol{H}\nabla)\boldsymbol{V} = \eta\nabla^2\boldsymbol{H}, \quad (2.3)$$

$$\nabla\boldsymbol{V} = 0, \quad (2.4)$$

$$\nabla\boldsymbol{H} = 0, \quad (2.5)$$

where ρ_0 is the constant reference density, $\boldsymbol{X}$ = (0, 0, $-g$) the acceleration due to gravity, $\boldsymbol{\Omega}$ = (0, 0, Ω), the angular velocity, $\boldsymbol{H}$ = (H_1, H_2, H_3) the intensity of the magnetic field, ν the kinematic viscosity, κ the thermal diffusivity, ρ the fluid density, α the coefficient of volume expansion, μ the magnetic permeability. $\eta = 1/4\pi\mu\sigma$ is the resistivity of the fluid, while σ is the electrical conductivity. $\boldsymbol{V} = (u, v, w)$, p and T are, respectively, the fluid velocity, the pressure and the temperature, while t is the time. The relationship between ρ and T is given by

$$\rho = \rho_0[1-\alpha(T-T_0)]. \quad (2.6)$$

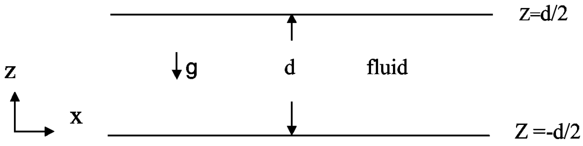

Fig. 1. Benard configuration.

For temperature modulation, we write the following externally imposed boundary conditions:

(i) When the temperature of the lower boundary as well as of the upper boundary is modulated, we have

$$\begin{aligned} T(t) &= \beta d(1+\varepsilon\cos\omega t) \;\text{ at } z=-d/2 \\ &= \beta d\varepsilon\cos(\omega t+\phi) \;\text{ at } z=d/2. \end{aligned} \quad (2.7)$$

(ii) When the upper boundary is held at constant temperature and only the lower boundary temperature is modulated, then

$$\begin{aligned} T(t) &= \beta d(1+\varepsilon\cos\omega t) \;\text{ at } z=-d/2, \\ &= 0 \qquad\qquad\qquad\;\; \text{ at } z=d/2, \end{aligned} \quad (2.8)$$

where β is the thermal gradient, ε represents the amplitude of modulation and ϕ is the phase angle. The equilibrium solution of the above system is

$$\boldsymbol{V} = (u,v,w) = 0, \quad T = \bar{T}(z,t), \quad p = \bar{p}(z,t). \quad (2.9)$$

The temperature $\bar{T} = T(z,t)$ is given by the diffusion equation

$$\frac{\partial \bar{T}}{\partial t} = \kappa\frac{\partial^2 \bar{T}}{\partial z^2}. \quad (2.10)$$

The differential equation (2.10) is solved for both the above cases (i) and (ii). We write

$$\bar{T}(z,t) = T_S(z) + \varepsilon\,\mathrm{Re}\{T_1(z,t)\}, \quad (2.11)$$

where $T_S(z)$ is the steady temperature field and T_1 is the oscillating part. Then we have

$$T_S(z) = \Delta T\left(\frac{1}{2}-\frac{z}{d}\right), \quad (2.12)$$

and

$$T_1(z,t) = \frac{\Delta T}{\sinh\lambda}\left\{ e^{i\phi}\sinh\lambda\left(\frac{1}{2}+\frac{z}{d}\right) + \sinh\lambda\left(\frac{1}{2}-\frac{z}{d}\right)\right\} e^{i\omega t}, \quad (2.13)$$

where

$$\lambda^2 = \mathrm{i}\omega d^2/\kappa. \tag{2.14}$$

If the system (2.9) is slightly perturbed, then we have

$$\begin{aligned} &\boldsymbol{V} = (u,v,w), \quad T = \bar{T}(z,t) + \theta, \\ &p = \bar{p}(z,t) + p_1, \quad \boldsymbol{H} = \boldsymbol{H}_\tau + \boldsymbol{h}. \end{aligned} \tag{2.15}$$

A constant magnetic field $H = |\boldsymbol{H}|$ is maintained, externally, in the direction of a unit vector $\boldsymbol{\tau}$, $\boldsymbol{h}$ is the perturbation in the magnetic field caused by the motions and the induced currents in the field. We substitute (2.15) into (2.1)–(2.5) and linear with respect to the perturbation quantities $\boldsymbol{V}$, θ and $\boldsymbol{h}$. For solution we Fourier analyze the disturbances, in the xy-plane and write

$$w(x,y,z,t) = w(z,t)\exp[\mathrm{i}(a_x x + a_y y)], \tag{2.16}$$

$$\theta(x,y,z,t) = \theta(z,t)\exp[\mathrm{i}(a_x x + a_y y)], \tag{2.17}$$

$$h_z(x,y,z,t) = h_z(z,t)\exp[\mathrm{i}(a_x x + a_y y)], \tag{2.18}$$

$$\zeta(x,y,z,t) = \zeta(z,t)\exp[\mathrm{i}(a_x x + a_y y)], \tag{2.19}$$

$$\xi(x,y,z,t) = \xi(z,t)\exp[\mathrm{i}(a_x x + a_y y)], \tag{2.20}$$

where h_z, ζ, ξ are the vertical component of the perturbed magnetic field $\boldsymbol{h}$, the vorticity, and the current density, respectively. We consider that the magnetic field is in vertical direction, and so $\tau = (0,0,1) = \hat{\boldsymbol{k}}$ (unit vector in vertical direction). Here $a = (a_x^2 + a_y^2)^{1/2}$ is the horizontal wavenumber. If we scale length, time, temperature, wavenumber, velocity and pressure according to

$$\begin{aligned} &z = dz', \quad t = t'/\omega, \quad \bar{T} = \beta d\hat{T}, \theta = \beta d\theta', \\ &a^2 = d^2 a'^2 \quad \boldsymbol{V} = (\alpha g \beta d^3 a'^2/\nu)\boldsymbol{V}', \\ &p_1 = (\alpha g \beta \kappa d^2 \rho_0/\nu)p', \quad \boldsymbol{h} = H\boldsymbol{h}', \end{aligned} \tag{2.21}$$

then the governing equations in linear form are

$$\begin{aligned} \omega^* \frac{\partial \boldsymbol{V}}{\partial t} + \frac{1}{a^2\rho_0}\nabla p = \frac{P_1}{a^2}\theta\hat{\boldsymbol{k}} + P_1\nabla^2\boldsymbol{V} - \sqrt{T}P_1(\hat{\boldsymbol{k}}\times\boldsymbol{V}) \\ + \frac{P_1P_2Q}{a^2R}(\hat{\boldsymbol{k}}\nabla)\boldsymbol{h}, \end{aligned} \tag{2.22}$$

$$\omega^* \frac{\partial\theta}{\partial t} + a^2R\left(\frac{\partial\hat{T}}{\partial z}\right)w = \nabla^2\theta, \tag{2.23}$$

$$\omega^* \frac{\partial\boldsymbol{h}}{\partial t} + a^2R(\hat{\boldsymbol{k}}\boldsymbol{\nabla})\boldsymbol{V} = P_2\nabla^2\boldsymbol{h}, \tag{2.24}$$

$$\boldsymbol{\nabla}\boldsymbol{V} = 0, \tag{2.25}$$

$$\boldsymbol{\nabla}\boldsymbol{h} = 0, \tag{2.26}$$

where $P_1 = \nu/\kappa$ is the Prandtl number, $P_2 = \nu/\eta$ is the magnetic Prandtl number, $R = \alpha g\Delta T d^3/\nu\kappa$ is the Rayleigh number, $T = 4\Omega^2 d^4/\nu^2$ is the Taylor number, $Q = \mu H^2 d^2/4\pi\rho_0\nu\eta$ is the Chandrasekhar number and $\omega^* = \omega d^2/\kappa$ is the non-dimensional frequency. In the above equations the primes have been omitted. The temperature gradient $\partial\hat{T}/\partial z$, obtained from the dimensionless form of (2.11) is

$$\frac{\partial\hat{T}}{\partial z} = -1 + \varepsilon\mathrm{Re}[g(z)\mathrm{e}^{\mathrm{i}t}], \tag{2.27}$$

where

$$g(z) = \frac{\lambda}{\sinh\lambda}\left\{\mathrm{e}^{\mathrm{i}\phi}\cosh\lambda\left(\frac{1}{2}+z\right) - \cosh\lambda\left(\frac{1}{2}-z\right)\right\}$$

and

$$\lambda^2 = \mathrm{i}\omega^*.$$

Henceforth the asterisk will be dropped and ω will be considered as the non-dimensional frequency. Now taking the curl of (2.22) twice, and using (2.16)–(2.20) we get the system of equations as

$$\begin{aligned} \omega(D^2 - a^2)\frac{\partial w}{\partial t} = P_1(D^2-a^2)^2 w - P_1\theta \\ + \frac{P_1P_2Q}{a^2R}(D^2-a^2)\frac{\partial h_z}{\partial z} \\ - \sqrt{T}P_1 D\zeta, \end{aligned} \tag{2.28}$$

$$\omega\frac{\partial\theta}{\partial t} = a^2R[1 - \varepsilon\mathrm{Re}\{g(z)\mathrm{e}^{\mathrm{i}t}\}]w + (D^2-a^2)\theta, \tag{2.29}$$

$$\omega\frac{\partial h_z}{\partial t} = a^2R\frac{\partial w}{\partial z} + P_2(D^2-a^2)h_z, \tag{2.30}$$

$$\omega\frac{\partial\zeta}{\partial t} = \sqrt{T}P_1\frac{\partial w}{\partial z} + P_1(D^2-a^2)\zeta + \frac{P_1P_2Q}{a^2R}\frac{\partial\xi}{\partial z}, \tag{2.31}$$

$$\omega\frac{\partial\xi}{\partial t} = a^2R\frac{\partial\zeta}{\partial z} + P_2(D^2-a^2)\xi, \tag{2.32}$$

where $D \equiv \frac{\partial}{\partial z}$. The boundary conditions for the rigid and electrically conducting walls are given by

$$w = Dw = \theta = 0 \;\text{ on }\; z = \pm\frac{1}{2}, \tag{2.33}$$

$$h_z = \zeta = D\xi = 0 \;\text{ on }\; z = \pm\frac{1}{2}. \tag{2.34}$$

3. Method

Using Galerkin technique, we transform the partial differential equations (2.28)–(2.32) into a system of ordinary differential equations. The latter are then solved numerically. We put

$$w(z,t) = \sum_{m=1}^{N} A_m(t)\psi_m(z), \tag{3.1}$$

$$\theta(z,t) = \sum_{m=1}^{N} B_m(t)\varphi_m(z), \tag{3.2}$$

$$h_z(z,t) = \sum_{m=1}^{N} C_m(t)\phi_m(z), \tag{3.3}$$

$$\zeta(z,t) = \sum_{m=1}^{N} D_m(t)\zeta_m(z), \tag{3.4}$$

$$\xi(z,t) = \sum_{m=1}^{N} E_m(t)\xi_m(z), \tag{3.5}$$

where

$$\psi_m(z) = \begin{cases} \dfrac{\cosh\mu_m z}{\cosh\frac{\mu_m}{2}} - \dfrac{\cos\mu_m z}{\cos\frac{\mu_m}{2}} & \text{if } m \text{ is odd,} \\ \dfrac{\sinh\mu_m z}{\sinh\frac{\mu_m}{2}} - \dfrac{\sin\mu_m z}{\sin\frac{\mu_m}{2}} & \text{if } m \text{ is even,} \end{cases} \tag{3.6}$$

$$\varphi_m(z) = \sqrt{2}\sin m\pi\left(z+\frac{1}{2}\right), \tag{3.7}$$

$$\phi_m(z) = \zeta_m(z) = \sqrt{2}\sin\left[(m+1)\pi z + (m-1)\frac{\pi}{2}\right], \tag{3.8}$$

$$\xi_{2m-1}(z) = \frac{\cosh mz}{\sinh\frac{m}{2}} + \frac{\cos mz}{\sin\frac{m}{2}}, \tag{3.9}$$

$$\xi_{2m}(z) = \frac{\sinh mz}{\cosh\frac{m}{2}} - \frac{\sin mz}{\cos\frac{m}{2}}, \quad (m = 1,2,3,\ldots). \tag{3.10}$$

The functions $\psi_m(z)$, $\varphi_m(z)$, $\phi_m(z)$ and $\zeta_m(z)$, above are defined in a manner so that they vanish at $z = \pm\frac{1}{2}$. Also clearly the derivatives of $\xi_m(z)$ vanish at the boundaries. For the derivatives of $\psi_m(z)$ to vanish at these boundaries, it is required that μ_m are to be the roots of the characteristic equation

$$\tanh\frac{1}{2}\mu_m - (-1)^m\tan\frac{1}{2}\mu_m = 0. \tag{3.11}$$

The roots of the equations (3.11) are given in the book of Chandrasekhar ([5], p. 636). Now we substitute (3.1)–(3.5) into (2.28)–(2.32) and then multiply these equations by $\psi_n(z)$, $\varphi_n(z)$, $\phi_n(z)$, $\zeta_n(z)$ and $\xi_n(z)$, respectively ($n = 1,2,3,\cdots,N$). The resulting equations are then integrated with respect to z in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$. The outcome is a system of $L = 5N$, ordinary differential equations for the unknown coefficients $A_n(t)$, $B_n(t)$, $C_n(t)$, $D_n(t)$ and $E_n(t)$:

$$\begin{aligned} &\omega\sum_{m=1}^{N}[K_{nm} - a^2\delta_{nm}]\frac{\mathrm{d}A_m}{\mathrm{d}t} \\ &= P_1\sum_{m=1}^{N}[(\mu_m^4 + a^4)\delta_{nm} - 2a^2K_{nm}]A_m - P_1\sum_{m=1}^{N}P_{nm}B_m \\ &\quad - \frac{P_1P_2Q}{a^2R}\sum_{m=1}^{\infty}[(m+1)^2\pi^2 + a^2]L_{nm}C_m \\ &\quad - \sqrt{T}P_1\sum_{m=1}^{\infty}S_{nm}D_m, \end{aligned} \tag{3.12}$$

$$\omega\frac{\mathrm{d}B_n}{\mathrm{d}t} = a^2R\sum_{m=1}^{N}[P_{mn} - \varepsilon\mathrm{Re}\{F_{nm}\mathrm{e}^{\mathrm{i}t}\}]A_m - (n^2\pi^2 + a^2)B_n, \tag{3.13}$$

$$\omega\frac{\mathrm{d}C_n}{\mathrm{d}t} = a^2R\sum_{m=1}^{N}R_{nm}A_m - P_2[(n+1)^2\pi^2 + a^2]C_n, \tag{3.14}$$

$$\begin{aligned} \omega\frac{\mathrm{d}D_m}{\mathrm{d}t} &= \sqrt{T}P_1\sum_{m=1}^{\infty}R_{nm}A_m \\ &\quad - P_1\left[(n+1)^2\pi^2 + a^2\right]D_n \\ &\quad + \frac{P_1P_2Q}{a^2R}\sum_{m=1}^{\infty}T_{nm}E_m, \end{aligned} \tag{3.15}$$

$$\begin{aligned} \omega\sum_{m=1}^{N}U_{nm}\frac{\mathrm{d}E_m}{\mathrm{d}t} &= a^2R\sum_{m=1}^{N}V_{nm}D_m \\ &\quad + P_2\sum_{m=1}^{N}[W_{nm} - a^2U_{nm}]E_m \end{aligned} \tag{3.16}$$

$(n = 1,2,\cdots,N)$,

where δ_{nm} is the Kronecker delta. The other coefficients, which occur in (3.12)–(3.16) are

$$K_{nm} = \int_{-1/2}^{1/2} D^2\psi_m(z)\psi_n(z)\mathrm{d}z, \tag{3.17}$$

$$P_{nm} = \int_{-1/2}^{1/2} \varphi_m(z)\psi_n(z)\mathrm{d}z, \tag{3.18}$$

$$L_{nm} = \int_{-1/2}^{1/2} D\phi_m(z)\psi_n(z)\mathrm{d}z, \tag{3.19}$$

$$S_{nm} = \int_{-1/2}^{1/2} D\zeta_m(z)\psi_n(z)\mathrm{d}z, \tag{3.20}$$

$$R_{nm} = \int_{-1/2}^{1/2} D\psi_m(z)\phi_n(z)\mathrm{d}z, \tag{3.21}$$

$$T_{nm} = \int_{-1/2}^{1/2} D\xi_m(z)\zeta_n(z)\mathrm{d}z, \tag{3.22}$$

$$U_{nm} = \int_{-1/2}^{1/2} \xi_m(z)\xi_n(z)\mathrm{d}z, \tag{3.23}$$

$$V_{nm} = \int_{-1/2}^{1/2} D\zeta_m(z)\xi_n(z)\mathrm{d}z, \tag{3.24}$$

$$W_{nm} = \int_{-1/2}^{1/2} D^2\xi_m(z)\xi_n(z)\mathrm{d}z, \tag{3.25}$$

and

$$F_{nm} = \int_{-1/2}^{1/2} g(z)\psi_m(z)\varphi_n(z)\mathrm{d}z. \tag{3.26}$$

The integrals (3.17) – (3.26) have been calculated numerically using Simpson's (1/3)rd rule [19]. For computational purposes it is convenient to rearrange the equations (3.12) – (3.16). For this, first we multiply (3.12) and (3.16) by the inverse of the matrices $[K_{nm} - a^2\delta_{nm}]$ and $[U_{nm}]$, respectively, and then introduce the notation

$$\begin{aligned} &x_1 = A_1, \quad x_2 = B_1, \quad x_3 = C_1, \quad x_4 = D_1, \\ &x_5 = E_1, \quad x_6 = A_2, \quad x_7 = B_2, \quad \ldots \end{aligned} \tag{3.27}$$

Now the equations (3.12) – (3.16) can be written in the form

$$\begin{aligned} &\frac{\mathrm{d}x_i}{\mathrm{d}t} = G_{i1}x_1 + G_{i2}x_2 + \cdots + G_{iL}x_L \\ &(i = 1,2,3,\ldots,5N). \end{aligned} \tag{3.28}$$

Since the coefficients $G_{ij}(t)$ are periodic in t, the stability of the solution of (3.28) can be discussed on the basis of the classical Floquet theory [20]. Let

$$\begin{aligned} &x_n(t) = x_{in}(t) = \mathrm{col}\,[x_{1n}(t), x_{2n}(t), \ldots, x_{Ln}(t)] \\ &(n = 1,2,3,\ldots,5N) \end{aligned} \tag{3.29}$$

be the solutions of (3.28) which satisfy the initial conditions

$$x_{in}(0) = \delta_{in}. \tag{3.30}$$

The solutions (3.29) with the conditions (3.30) form $5N$ linearly independent solutions of (3.28). These solutions are found by integrating the system (3.28) using the Runge-Kutta-Gill procedure [19]. Then arrange them in the constant matrix

$$C = [x_{in}(2\pi)]. \tag{3.31}$$

Now using the Rutishauser method [21] we find the eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_L$ of the matrix C and then define the numbers μ_r, the characteristic exponents of C, by the relations

$$\lambda_r = \exp(2\pi\mu_r), \quad (r = 1,2,3,4,\ldots,5N). \tag{3.32}$$

The values of the characteristic exponents determine the stability of the system. We assume that the μ_r are ordered so that

$$\mathrm{Re}(\mu_1) \geq \mathrm{Re}(\mu_2) \geq \ldots \geq \mathrm{Re}(\mu_L). \tag{3.33}$$

Then the system is stable if $\mathrm{Re}(\mu_1) < 0$, while $\mathrm{Re}(\mu_1) = 0$ corresponds to one periodic solution and represents a stability boundary. This periodic disturbance is the only disturbance, which will manifest itself at marginally stability.

4. Results and Discussion

Here in the Galerkin procedure, we determine the value of N in the process of numerical solution. It is found that for the parameter ranges of our interest it is sufficient to take $N = 2$ (two Galerkin terms – one even and one odd). Therefore all the following results are related to $N = 2$, i. e. a system of 10 ordinary differential equations have been solved. The critical values of the Rayleigh number and the wavenumber in the absence of modulation ($\varepsilon = 0$) are found as follows:

$$T = 0.0,\ Q = 0.0,\ Rc = 1728.38543,\ a_{\mathrm{c}} = 3.1. \tag{4.1}$$

The result (4.1) corresponds to non-rotating, non-magneto convection cases (Rosenblat and Tanaka [14] and Bhadauria and Bhatia [18]). On comparing the results of Tables 1 and 2 with (4.1), it is clear that the effect of a rotation and magnetic field is to stabilize the system, as convection is occurring at higher Rayleigh

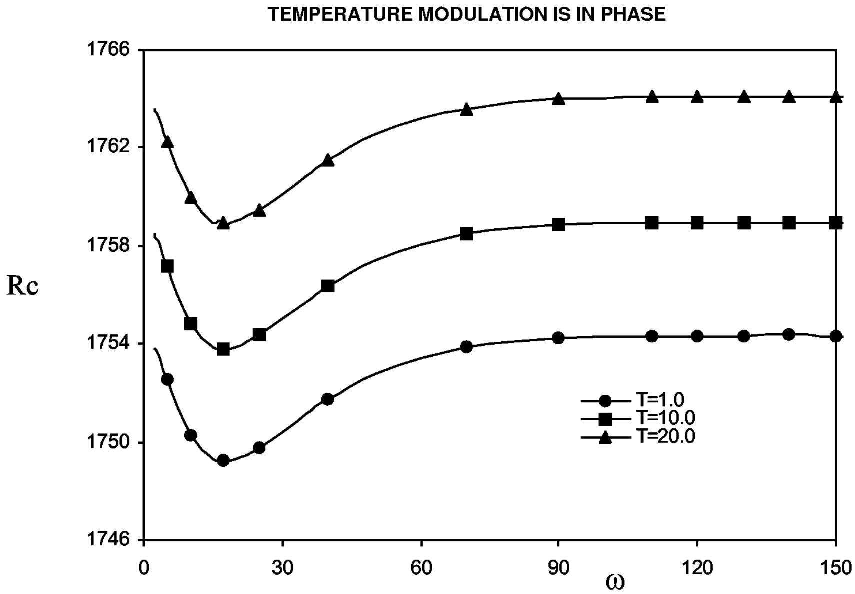

Fig. 2. Variation of Rc with ω; $Q = 1.0$, $\varepsilon = 0.5$, $P_1 = 1.0$, $P_2 = 1.0$.

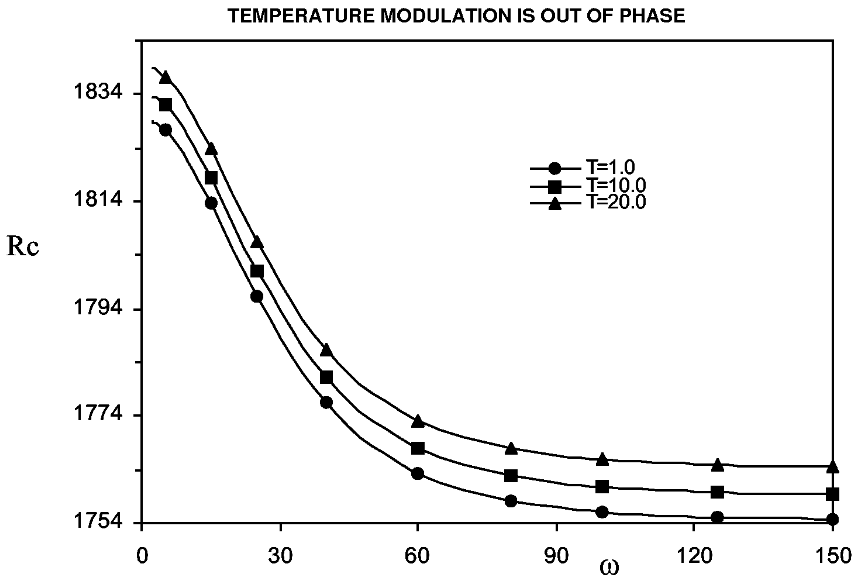

Fig. 3. Variation of Rc with ω; $Q = 1.0$, $\varepsilon = 0.5$, $P_1 = 1.0$, $P_2 = 1.0$.

Table 1. $Q = 1.0$.

	$T = 0.0$	$T = 1.0$	$T = 10.0$	$T = 20.0$
Rc	1753.7656	1754.2805	1758.9092	1764.0425
a_c	3.113	3.114	3.118	3.122

Table 2. $T = 10.0$.

	$Q = 0.0$	$Q = 0.1$	$Q = 0.5$	$Q = 1.0$
Rc	1733.5653	1736.1046	1746.2529	1758.9092
a_c	3.1	3.1038	3.11	3.118

numbers. This confirms the results of Aurnou and Olson [11] and Dhiman Joginder [10] that rotation and magnetic fields have a stabilizing effect on the onset of thermal instability.

Now we find the effect of modulation by considering $\varepsilon \neq 0$, and calculate the modified value of Rc, with variation in other parameters. We also check the critical value of the wavenumber a. The values of Rc have been calculated for the following three cases:

a) when the plate temperatures are modulated in phase, i. e. $\phi = 0$,

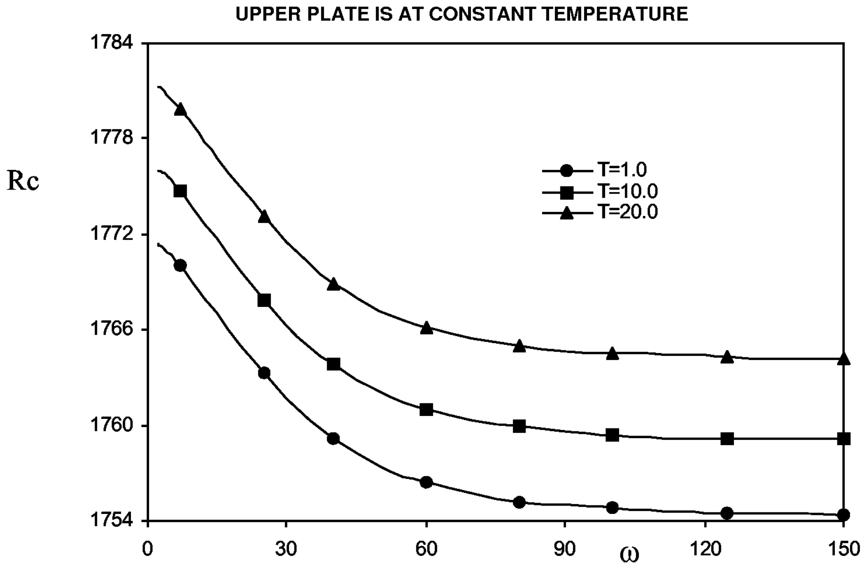

Fig. 4. Variation of Rc with ω; $Q = 1.0$, $\varepsilon = 0.5$, $P_1 = 1.0$, $P_2 = 1.0$.

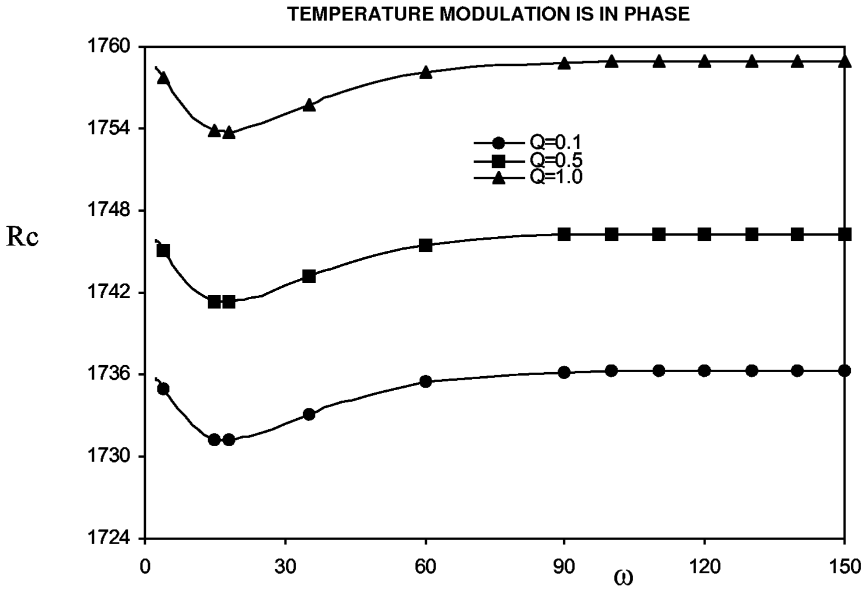

Fig. 5. Variation of Rc with ω; $T = 10.0$, $\varepsilon = 0.5$, $P_1 = 1.0$, $P_2 = 1.0$.

b) when the plate temperatures are modulated out of phase, i. e. $\phi = \pi$, and

c) when only the bottom plate temperature is modulated, the upper plate is held at a fixed constant temperature.

In Figs. 2–4, we depict the variation of Rc with ω, for all the three cases and for different values of the Taylor number T, other parameters are held constant, $Q = 1.0$, $P_1 = P_2 = 1.0$, $\varepsilon = 0.5$. Figures 5–7 show the variation of Rc with ω, for different values of the Chandrasekhar number Q, other parameters are $T = 10.0$, $P_1 = P_2 = 1.0$, $\varepsilon = 0.5$. From the Figs. 2 and 5 we see that for in-phase modulation, the effect of modulation is to destabilize the system with convection occurring at an earlier point than in the steady temperature gradient case. This effect of modulation is small when ω is small and falls off to zero as $\omega \to \infty$. For intermediate values of ω the effect of modulation is greatest near $\omega = 17$. This agrees with the results of Bhadauria and Bhatia [18]. Also this agrees with Venezian [13], in which the peak negative value of Rc was obtained near $\omega = 20$.

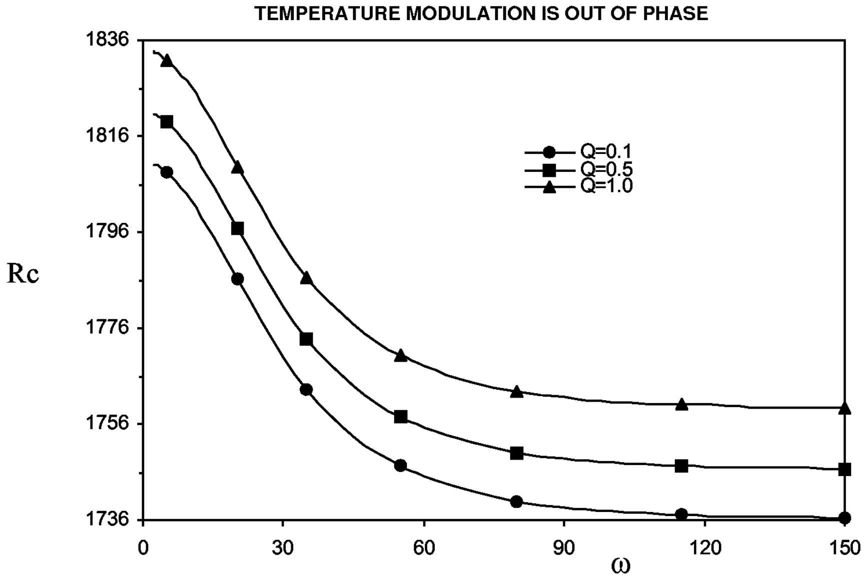

Fig. 6. Variation of Rc with ω; $T = 10.0$, $\varepsilon = 0.5$, $P_1 = 1.0$, $P_2 = 1.0$.

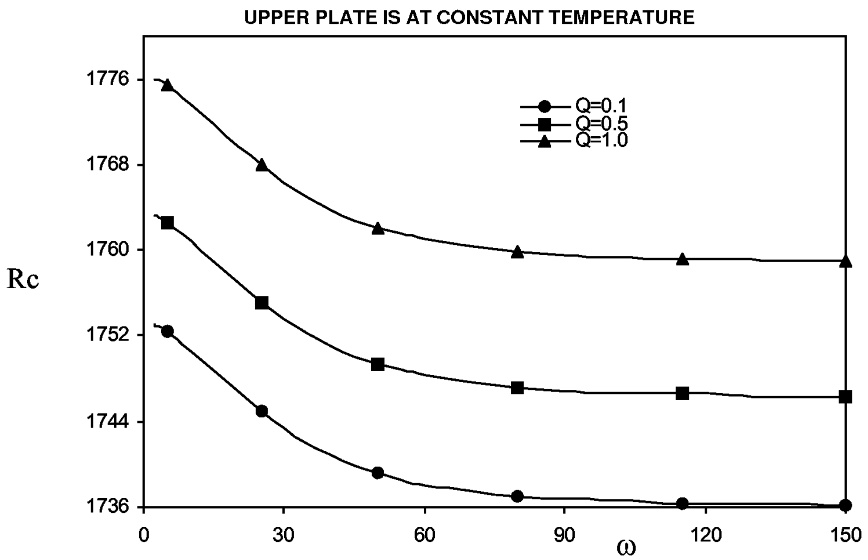

Fig. 7. Variation of Rc with ω; $T = 10.0$, $\varepsilon = 0.5$, $P_1 = 1.0$, $P_2 = 1.0$.

When the temperature modulation is out of phase or the upper plate is at constant temperature, the effect is stabilization, decreasing with increasing frequency ω (Figs. 3, 4, 6, 7). The stabilization is greatest near $\omega = 0$ and disappears altogether when the frequency ω is sufficiently large. This agrees with the results of Bhadauria and Bhatia [18], Bhatia and Bhadauria [17], Rosenblat and Tanaka [14] and Venezian [13]. Here at low frequency the convective wave propagates across the fluid layer, thereby inhibiting the instability, and so the convection occurs at higher Rayleigh number than that predicted by the linear theory with steady temperature gradient [(4.1), Tables 1 and 2].

When the frequency of modulation is small, the effect of modulation is felt throughout the fluid layer. If the modulation is in phase, the temperature profile consists of the steady straight-line section plus a parabolic profile that oscillates with time. As the amplitude of the modulation increases, the parabolic part of the profile becomes more and more significant. It is known that a parabolic profile is subject to finite-amplitude instabilities (Krishnamurti [22], Veronis [23]) so that

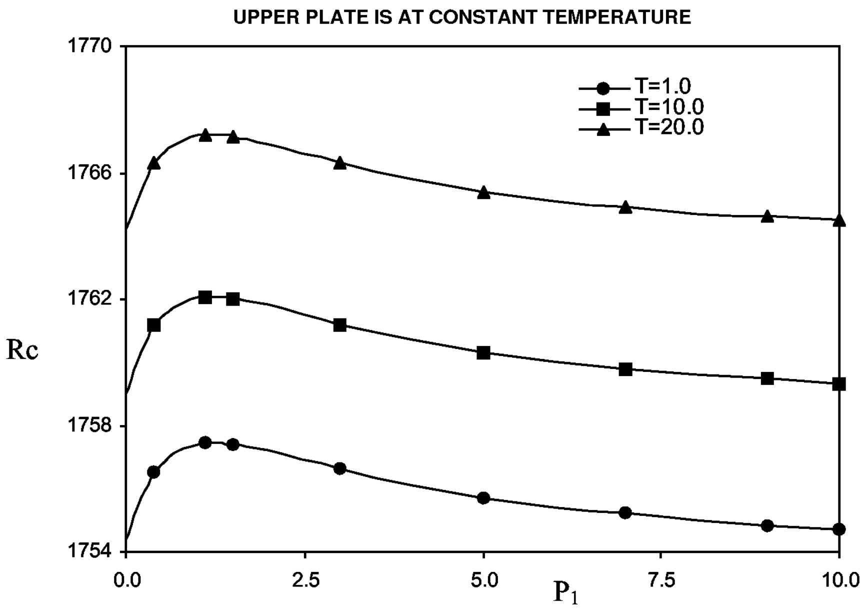

Fig. 8. Variation of Rc with P_1; $Q = 1.0$, $\varepsilon = 0.5$, $\omega = 50.0$, $P_2 = 1.0$.

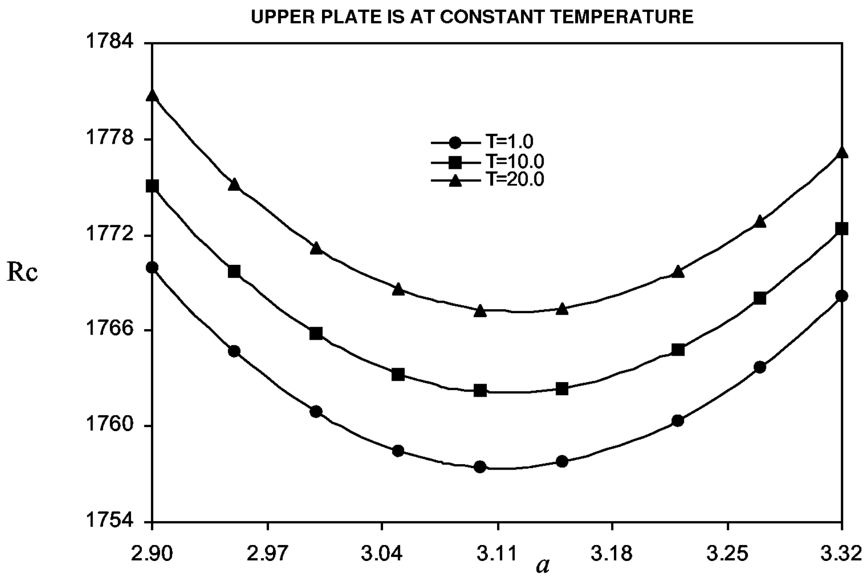

Fig. 9. Variation of Rc with a; $Q = 1.0$, $\omega = 50.0$, $\varepsilon = 0.5$, $P_1 = 1.0$, $P_2 = 1.0$.

convection occurs at lower Rayleigh number than that predicted by the linear theory with steady temperature gradient (Figs. 2, 5).

From the Figs. 2 – 4 and 5 – 7 we see that the values of the critical Rayleigh number approach the zero modulation values, given in Tables 1 and 2, respectively. Thus the effect of temperature modulation becomes almost negligible at large values of ω. At high frequency, since the modulation becomes very fast, the temperature in the fluid layer is unaffected by the modulation except for a thin layer, so that we find almost the same value of Rc as for zero modulation.

Figure 8 shows the variation of Rc with P_1 [for case (c)], at $Q = 1.0$, $\varepsilon = 0.5$, $\omega = 50.0$, $P_2 = 1.0$ for different values of T. From the figure we see that the effect of modulation is maximum at around $P_1 = 1.0$, and then falls off to zero when P_1 increases further. This agrees with Rosenblat and Tanaka [14].

In Fig. 9, we have depicted the variation of Rc with the wavenumber a, to check the value of the critical

Rayleigh number. It is very clear from the figure that for all the three graphs considered, the critical value of the wavenumber *a* is near 3.1, same as in the Tables 1 and 2.

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